NOTATION

T, temperature; v, velocity; p, pressure; ν, coefficient of kinematic viscosity; η, coefficient of dynamic viscosity; χ, thermal diffusivity; $\varkappa$, thermal conductivity; r, θ, φ, coordinates; a, bubble radius; A, constant temperature gradient; u, drift velocity; 2H, average curvature of surface; M, M*, P, α_0, dimensionless parameters of problem; differentiation with respect to the coordinate θ is denoted by a prime; P_l, Legendre polynomials of order l; α, coefficient of surface tension; $\mathbf{r}_1$, $\vec{\theta}_1$, unit vectors of spherical coordinate system.

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VISCOSITY OF A WATER-FLUIDIZED BED

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The method of damping of oscillations of a ball submerged in a fluidized bed is used to study the viscosity of the bed.

The rheological properties of an air-fluidized bed have been studied rather extensively and by different methods. There are experimental data which we obtained [1, 2] allowing one to draw certain conclusions concerning the dependence of the effective viscosity of the bed on the properties of the solid phase.

To clarify the mechanism of the effective viscosity and the laws of its variation it is necessary to study beds fluidized by different agents, and therefore it is advisable to make measurements in a bed fluidized by water. These measurements present definite difficulties, since in its rheological properties a strongly rarefied bed approaches the properties of the fluidizing agent, the viscosity of which is low. Using the method which we developed [3], which provides for the motion of bodies in the bed in the region of small Reynolds

TABLE 1. Characteristics of Substances Used for Calibration

Substance, % at t, °C	$\rho \cdot 10^{-3}$, kg/m³	$\mu \cdot 10$, N·sec/m²	N_{exp}	$\nu \cdot 10^4$, m²/sec	$N_{exp}\,\rho \cdot 10^{-3}$, kg/m³
Water at 20	1,0	$1{,}005 \cdot 10^{-2}$	55,5	0,01	55,5
Aqueous solution of sugar					
20 at 21	1,08	$1{,}96 \cdot 10^{-2}$	40	0,018	43,20
40 at 20	1,18	$6{,}2 \cdot 10^{-2}$	25,8	0,053	30,44
60 at 34	1,29	$27{,}97 \cdot 10^{-2}$	14	0,217	18,06
60 at 30	1,29	$33{,}78 \cdot 10^{-2}$	13,2	0,262	17,03
60 at 25	1,29	$43{,}86 \cdot 10^{-2}$	11,0	0,340	14,19
60 at 20	1,29	$56{,}5 \cdot 10^{-2}$	9,5	0,438	12,25
Glycerin	1,24	3,68	4	2,968	4,96
Castor oil	0,95	9,03	2	9,505	1,90

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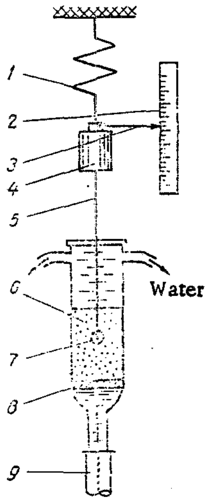

Fig. 1

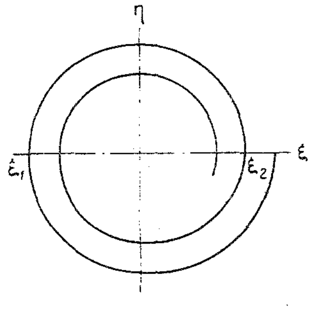

Fig. 2

Fig. 1. Diagram of experimental setup: 1) spring; 2) scale; 3) pointer; 4) cylindrical body; 5) rod; 6) bed; 7) ball; 8) filter; 9) flowmeter.

Fig. 2. Graph of dependence of η on ξ.

numbers Re, one can reliably determine the viscosity of the bed if the fluidizing agent is air, whose density ρ_0 and viscosity μ_0 are low, from the ratio $\mu_0/\rho_0 \simeq 10^{-5}$ m^2/sec. For water μ_0/ρ_0 is an order of magnitude smaller than for air.

Wishing to make it so that the body (a ball) moving in the bed not only disrupts the structure of the bed to the minimum but also reacts weakly to the pulsation impacts characteristic of the bed, one must choose a body of small volume (with a small surface) and large mass. But then because of the high velocity of fall values of Re < 1 will not be provided and the main condition for the mode of viscous flow will be violated.

Vibrational viscometers [4], in which a flat plate submerged in the medium under study undergoes harmonic oscillations with a small amplitude under the effect of a harmonic driving force, can be used to measure the viscosity of true liquids. The quality of the mechanical vibrational system of such instruments is rather high; moreover, since the amplitude of the vibrations is only several dozen microns, the velocity of the plate motion is low, the critical Reynolds number is not reached (for a plate $\mathrm{Re}_{\mathrm{cr}} \approx 10^5$), and its vibrations remain in the viscous region. This system cannot be applied directly for measuring the viscosity of a fluidized bed because of the considerable intrinsic pulsations of the bed. The method can be modified somewhat, however.

Let us assume that an oscillatory system of mass M, consisting of a spring and a thin rod with a ball of diameter d fastened to it and submerged in a medium of density ρ and viscosity μ, undergoes natural damped oscillations (Fig. 1). Since it is possible that the mode of oscillations will not be purely viscous, let us analyze the equation of motion of such a system:

$$M\frac{dv}{dt} = F_{\mathrm{res}} + F_{\mathrm{el}}. \tag{1}$$

Allowing not only for the effect of forces of viscous friction, but also for the inertial resistances of the medium to the established motion, one can set [5]

$$F_{\mathrm{res}} = -3\pi d\mu v - 0.06\,\pi d^2\rho v\,|v|. \tag{2}$$

The elastic force is

$$F_{\mathrm{el}} = -M\omega_0^2 x = -kx. \tag{3}$$

Equation (1) can thus be rewritten in the form

$$M\frac{dv}{dt} + 3\pi d\mu v + 0.06\,\pi d^2\rho v\,|v| + M\omega_0^2 x = 0. \tag{4}$$

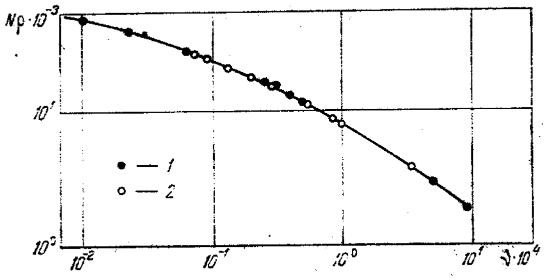

Fig. 3

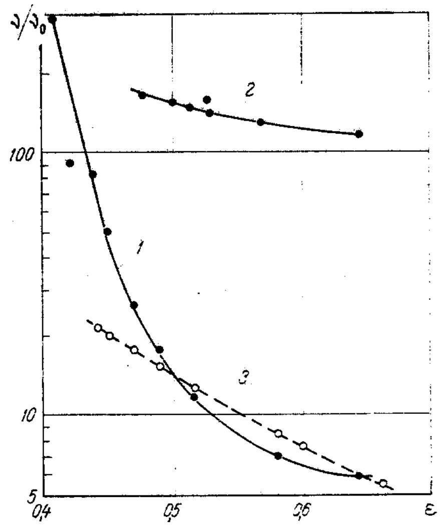

Fig. 4

Fig. 3. Graph of dependence of $N\rho$ on ν (calibration curve): 1) true liquids and solutions; 2) sand (0.3-0.5 mm) fluidized by water. $N\rho \cdot 10^{-3}$, kg/m^3; ν, m^2/sec.

Fig. 4. Graph of dependence of ν/ν_0 on ε: 1) fluidizing agent: water; 2) air; 3) from Eq. (21).

Since

$$\frac{dx}{dt} = v, \tag{5}$$

by dividing (4) by (5) we obtain

$$M\frac{dv}{dx} + 0.06\,\pi d^2\rho\,|v| + M\omega_0^2\frac{x}{v} + 3\pi d\mu = 0. \tag{6}$$

In the absence of resistance the system would undergo purely harmonic oscillations, passing through the equilibrium position with a maximum velocity

$$v_m = \sqrt{\frac{k}{M}}\,x_0. \tag{7}$$

In reality, the oscillations are damped. As a parameter characterizing the process we choose the number of oscillations N after which the amplitude x_0 of the oscillations decreases twofold. Let us return to the initial equation (6) and convert it to dimensionless form. We designate

$$\tau = \frac{v_m}{x_0}t,\ \frac{x}{x_0} = \xi,\ \frac{v}{v_m} = \eta,\ \mathrm{Re}_m = \frac{v_m d\rho}{\mu}. \tag{8}$$

We introduce the dimensionless quantity

$$Q = \frac{3\pi d^2\rho x_0}{M} \tag{9}$$

and reduce (6) to the form

$$\frac{d\eta}{d\xi} = -\frac{\xi}{\eta} - \frac{Q}{\mathrm{Re}_m}(1 + 0.02\,\mathrm{Re}_m|\eta|). \tag{10}$$

Equation (10) must be solved with the initial conditions

$$\eta = 0 \quad \text{at} \quad \xi = 1 \tag{11}$$

and certain limits of variation of Re_m and Q.

Let us estimate these limits on the basis of the concrete conditions of the experiment. In our setup (Fig. 1) a spring of mass 0.223 kg and elastic coefficient k = 13.75 N/m was fastened rigidly at one end in a special brace. At the other end of the spring was fastened a pendant consisting of a massive cylindrical body and a thin brass rod ending in a ball of diameter d = $1.67 \cdot 10^{-2}$ m; the mass of the pendant was 0.6855 kg.

TABLE 2. Viscosity of a Fluidized Sand Bed

Fluidizing agent: water						
$\rho \cdot 10^{-3}$, kg/m^3	ε	N	$N\rho \cdot 10^{-3}$, kg/m^3	$\nu \cdot 10^4$, m^2/sec	$\mu \cdot 10$, N·sec/m^2	$\frac{\nu}{\nu_0}$
1,966	0,411	2,0	3,93	3,25	6,39	325
1,946	0,423	4,0	7,78	0,92	1,79	92
1,917	0,441	4,4	8,43	0,84	1,61	84
1,897	0,453	6,2	11,76	0,51	0,97	51
1,862	0,474	9,4	17,50	0,26	0,48	26
1,838	0,489	10,8	19,85	0,18	0,33	18
1,792	0,517	13,0	23,30	0,12	0,21	12
1,692	0,578	15,6	26,39	0,07	0,12	7
1,582	0 645	19,0	30,06	0,06	0,09	6

Air				
$\rho \cdot 10^{-3}$, kg/m^3	ε	$\nu \cdot 10^4$ m^2/sec	$\mu \cdot 10$, N·sec/m^2	$\frac{\nu}{\nu_0}$
—	—	—	—	—
—	—	—	—	
1,373	0,480	22,0	30,2	166
1,312	0,503	21,00	27,5	158
1,278	0,516	20,5	26,2	154
1,251	0,526	21,8	27,2	164
1,238	0,531	19,3	23,9	145
1,132	0,571	17,4	19,7	131
0,932	0,647	15,8	15,7	119

A pointer connected with the system moved relative to a scale in the course of the oscillations. The system, displaced from the equilibrium position by a distance $x_0 = 4 \cdot 10^{-2}$ m, underwent damped oscillations with a period T = 1.5 sec. The corresponding estimate of the quantities Q and Re_m under our conditions gives

$$0.14 \leqslant Q \leqslant 0.28,\ 3 \leqslant \mathrm{Re}_m \leqslant 3000. \tag{12}$$

Now let us analyze Eq. (10) for the two extreme cases of motion of the system: a) the viscosity of the medium is high; b) the viscosity of the medium is low.

a) If μ is high, then one can neglect the term $0.02Q|\eta|$ in comparison with Q/Re_m in (10) and we obtain the differential equation for damped oscillations. The amplitude of the oscillations decreases by the law $\exp[-(Q/2\mathrm{Re}_m)\tau]$ and decreases twofold after a time $\tau_{1/2}$, having undergone $2\pi N$ oscillations:

$$N = \frac{\tau_{1/2}}{2\pi} = \frac{\ln 2 \cdot \mathrm{Re}_m}{\pi Q}. \tag{13}$$

b) If the viscosity μ of the medium is low, then we neglect the term Q/Re_m in (10). Introducing the designation

$$\frac{\eta^2}{2} = y \tag{14}$$

and transforming Eq. (10), we obtain

$$\frac{dy}{d\xi} \pm 0.04\, Qy = -\xi. \tag{15}$$

The solution of the nonhomogeneous equation (15) satisfying the initial conditions ($y = 0$ at $\xi = 1$) has the form

$$y = \frac{25}{Q}\left\{\frac{25}{Q} + \xi - \left(1 + \frac{25}{Q}\right)\exp\left[\frac{Q}{25}(\xi - 1)\right]\right\}. \tag{16}$$

Expanding $e^{Q/25(\xi-1)}$ in a series by powers of $(Q/25)(\xi - 1)$ and being limited to the first three terms of the series, we can estimate the value of ξ_1 at which y is reduced to zero (Fig. 2):

$$\xi_1 = e^{-\frac{2Q}{25}}. \tag{17}$$

After one full revolution ξ reaches the value $\xi_2 = e^{-4Q/25}$ and, consequently, the number N of revolutions after which the dimensionless amplitude decreases twofold is

$$N = \frac{25 \ln 2}{4Q}. \tag{18}$$

It is seen from Eqs. (13) and (18) that in both extreme cases the quantities N and Q enter into the solution in the form of a product, with NQ being a function of the Reynolds number when it is small and being independent of Re when it is large. Without performing the calculations one can in practice construct an experimental calibration curve from observations of the damping of oscillations in liquids whose viscosity and density are known. Since Q varies in proportion to the density ρ of the medium, while the Reynolds numbers vary in proportion to $1/\nu$, where ν is the kinematic viscosity of the medium, it is convenient to construct the calibration curve in the coordinates $N\rho - \log \nu$.

Data on the substances used for the calibration are presented in Table 1. The results of the experiments are represented graphically in Fig. 3.

The calibration curve later served for the determination of the viscosity of water-fluidized beds of granular materials.

A charge of the test material was poured into a column $7.4 \cdot 10^{-2}$ m in diameter. Water was supplied to the base of the column through a thin metal grid and a layer of felt (the filter) and the bed changed into a suspended state. The water flow rate was measured with an RS-5 flowmeter. The height of the bed and its porosity and viscosity varied as a function of the rate of supply of the water. The viscosity was determined from the number N of oscillations of the pendant in the bed and from the calibration graph (Fig. 3). The results of the measurement of the viscosity of a bed of sand with a size of 0.3-0.5 mm are presented in Table 2. Data which we obtained on the viscosity of a sand bed fluidized by air are also presented there for comparison.

The first theoretical work on the determination of the effective viscosity of dilute suspensions was the study of Einstein [6], who found that the change in the rate of settling of the solid particles can be explained by an increase in the viscosity μ of the suspension in comparison with the viscosity μ_0 of the medium:

$$\mu = \mu_0 (1 + 2.5\,\Omega), \tag{19}$$

where Ω is the portion of the volume of the suspension occupied by the solid phase. The Einstein equation was not confirmed experimentally for suspensions in which the concentration of solid particles exceeds 0.05. Subsequently many authors have proposed equations differing from (19) which are valid for narrow intervals of particle concentration. Thus, in [7], following an analysis of the experimental data of a number of studies on the settling of narrow fractions of sand, coal, and others, as well as data on beds fluidized by air and water, the following equation was proposed for calculating the apparent viscosity of a disperse system:

$$\mu = \frac{\mu_0}{\varepsilon^{1,285 + f^3 - 2(1-\varepsilon)(f-1)^{1/3}}} \cdot \frac{\rho}{\rho_0} = \mu_0' \frac{\rho}{\rho_0}, \tag{20}$$

where μ_0' is considered as the apparent viscosity of the disperse medium.

It follows from (20) that the ratios of the kinematic viscosities of the bed ν and the fluidizing agent ν_0 are determined by the porosity of the bed:

$$\frac{\nu}{\nu_0} = \frac{1}{\varepsilon^{1.285 + f^3 - 2(1-\varepsilon)(f-1)^{1/3}}}. \tag{21}$$

On the basis of our experimental data graphs of the dependence of ν/ν_0 on ε for a sand bed fluidized by water and air are constructed in Fig. 4 on a semilogarithmic scale. The curve corresponding to (21), which does not coincide with the experimental graphs, is also plotted there for comparison. From theoretical considerations it actually follows that a proportionality should exist between the kinematic viscosities of the bed and the fluidizing agent. Since the processes taking place in the bed are nonsteady, however, for appreciable expansions of the bed the connection between ν and ν_0 must evidently be different for the so-called heterogeneous fluidization when the fluidizing agent is water and for the heterogeneous fluidization when the fluidizing agent is air.

NOTATION

M, mass of oscillatory system; d, diameter of ball; ρ, μ, ν, density, dynamic viscosity, and kinematic viscosity of medium and of fluidized bed; ρ_0, μ_0, ν_0, density, dynamic viscosity, and kinematic viscosity of fluidizing agent; v, velocity of oscillatory motion of ball; v_m, maximum velocity; t, time of motion; F_{res}, force of resistance; F_{el}, elastic force; Re, Reynolds number; Re_m, Reynolds number corresponding to v_m; ω_0, frequency of natural undamped oscillations; x, displacement from equilibrium position; x_0, amplitude of oscillations; k, elastic coefficient, N, number of oscillations after which amplitude of oscillations decreases twofold; ξ, dimensionless displacement; η, dimensionless velocity; Q, dimensionless parameter; T, period of oscillations; τ, dimensionless time; $\tau_{1/2} = 2\pi N$; ξ_1, ξ_2, intermediate values of ξ.; $y = \eta^2/2$; Ω, portion of volume of suspension occupied by solid phase; ε, porosity of bed; f, coefficient of nonsphericity; $\mu^0 = \frac{\mu_0}{\varepsilon^{1.283} + f^3 - 2(1-\varepsilon)(f-1)^{1/3}}$.

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METHODS OF EXAMINING BEAM DIFFUSION IN AN ABSORBING AND SCATTERING MATERIAL

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UDC 536.3

An apparatus and method are presented for measuring the effective cross section of a radiation beam subject to reflection and transmission by layers of absorbing and scattering material.

Recent methods of measuring spectral characteristics for scattering materials subject to directional irradiation have made it necessary to make a detailed study of the propagation of a narrow parallel radiation beam in such a material; a particular feature here is that the beam is rapidly transformed to a purely diffuse beam on account of repeated scattering at optical nonuniformities [1-4]. The beam cross section increases considerably, and the multiple scattering makes a major contribution to the increase in cross section.

A study has been made [2] of the propagation of a narrow beam of light in a turbid medium having a highly elongated scattering indicatrix, and an analytical expression was derived for the effective radius of the beam r_{ef} in relation to the optical thickness. Results have been reported [3] on the radial dependence of the flux density after passage through small Lucite spheres (the measurements were made with the photocell and set of celluloid screens). Screens coated with graphite had clear rings of internal radius up to 6 mm. The main disadvantage of this method, which introduces an uncorrected error, is that the sensitivity of the photocell varies from part to part. On the other hand, these results [3] do define the radial dependence of the flux density. So far as we are aware, no study has been made of the radial dependence of the flux density for reflected fluxes.

We have examined this topic by means of special equipment whose major components were an adjustable iris diaphragm and a photometric sphere (Fig. 1).

The iris diaphragm had blackened metal blades of thickness 0.2 mm and allowed us to alter the diameter of the back-scattered and transmitted beam from 3 to 40 mm. The two fluxes were measured for a variety of

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